

A new and easy way of demonstrating some propositions in Euclid by the learned Mr. ----- Ash. a Member of the Philosophical Society of Dublin for promoting natural knowledge.

THE Preeminence of *Mathematical* knowledge, and the certainty of its way of reasoning are manifest from the few or no controversies between the *Professors* thereof (especially in pure unmixt *Mathematicks*;) and from the easy discovery of *paralogisms*. Some of the reasons of which certitude may be these: because *quantity*, the *object* about which it is conversant, is a sensible obvious thing, and consequently the *Ideas* we form thereof are clear and distinct, and dayly represented to us in most familiar instances; because it makes use of termes which are proper, adæquate, and unchangeable; its *axioms* and *postulata* also are very few and rational. It assigns such causes and generations of *Magnitudes* as are easily apprehended and readily admitted; it rejects all trifling in words and *Rhetorical schemes*, all conjectures, authorities, prejudices, and passion; Lastly so exquisite an order and method in *demonstrating* is observed, that no *proposition* is pretended to be proved, which does not plainly follow from what was before demonstrated, as is manifest in *Euclid's Elements*. Now as a farther instance of the evidence of *Mathematical Theoremes*, I believe it were not difficult to demonstrate any one of *Euclid's* independently from the rest, without precedent *Lemma's* or propositions; as an essay of which I will here subjoyn some of the most useful, and upon which the solution of most *problems*, especially *Algebraical* ones, do depend, and those also in the most various and different parts of *Geometry*, *viz.* concerning the properties of angles, circles, triangles, squares, proportionalls, and solids. The *Propositions* which I will endeavour to *demonstrate* thus independently

dently shall be these; the 32^d, and 47th, of the 1st book, most of the 2^d, and 5th books, the 1st, and 16th of the 6th, with their *Corollaries*. In order to demonstrate the 32^d, I suppose it known what is meant by an angle, triangle, circle, external angle, parallels, and that the measure of an angle is the arch of a circle intercepted between its sides, that a right angle is measured by a quadrant, and 2 right angles by a semicircle. I say then (in *Fig. 1.*) that in the triangle ABC, the external angle BCE is equal to the 2 opposite internal ones ABC, BAC; for let a circle be drawn, C being the Center, and BC the *radius*, and let CD be drawn parallel to AB, those 2 lines being always æquidistant will both have the same inclination to any 3^d line falling upon them, that is (by the definition of angle) they will make equal angles with it, for if any part of CD (for instance) did incline more to BC than did AB, upon that very account they would not be parallel, it follows therefore that the angles ABC, BCD are equal also BAC = DCE, because AE falls upon 2 parallels, but the external angle BCE = BCD + DCE which were before proved to be equal to ABC, BAC (Q. E. D.) hence may be infer'd as a *corollary*, that the 3 angles of every triangle are equal to 2 right ones, for the angles ACB + BCE are measured by a semicircle and therefore equal to 2 right angles, *Corollaries* also from hence are the 20th 22^d and 31st of the 3^d book which contain the properties of circles, whose deduction from hence being most natural and obvious, I omit.

In order to demonstrate the 47th, I suppose the meaning of the terms made use of to be known; and that 2 angles or superficies are equal when one being put on the other, it neither exceeds, nor is exceeded: this being allowed, I say the sides about the right angle are either equal or unequal, if equal (as in *Fig. 2.*) let all the squares be described, the whole figure exceeds the square of the *Hypotenuse* BC by the 2 triangles M, U, and exceeds also the squares of the other 2 sides AB. AC. by the 2 trian-

gles ABC, and S; which excesses are equal, for M is equal to ABC, the 2 sides about the right angle, being 2 sides of a square, upon AB by supposition equal to AC, and the 3^d side equal to BC, therefore the whole triangles are equal. after the same manner S and U are proved to be equal, therefore the square of CB is equal to the square of the 2 other sides QED.

But if the sides be unequal (as in *Fig. 3d*) let the square be described, and the *parallelogram* LQ completed, the whole Figure exceeds the square upon AC, by 3 triangles X, R, Z, and exceeds also the square LA, AD, by the triangle ABC, and the *Parallelogram* P. Q. which excesses I say are equal, for Z is equal to ABC, the side C=BC, CD=AC, the angle D=A, and OCD=BCA, which is manifest by taking the common angle ACO out of the 2 right angles BCO, ACD, therefore by superimposition the whole triangles are equal. In like manner X is proved equal to ABC, also R; and the parallelogram PQ to be double of the triangle ABC; thus the excesses being proved equal, the remainders also will be equal, *viz.* the square of BC to the square of AB, AC (Q. E. D) manifest *corollaries*, from hence are the 35th and 36th of the 3^d book, also the 12th and 13th of 2^d. And here I shall observe that by this *Method* of proving the 47. 1. *Eucl.* tis manifest that that proposition may be demonstrated otherwise then *Euclide* has done it, and yet without the help of proportions, which *Peletarius* denied as possible.

The first 10 propositions of the 2^d book are evidently demonstrated, only by substituteing *species* or letters instead of lines, and multiplying them according to the tenor of the proposition; thus to instance in one or two; in *Fig. 4* call the whole line A, and its parts B and C therefore $A=B+C$ and consequently $AA=BB+CC+2BC$ which is the very sense of the 4th of the 2^d book. Thus also (in *Fig. 5*) let a line be cut into equal parts F, F, and let another line S be added thereto, tis manifest that

$$4 FF + 4 SF + 2 SS = 2 FF + 2 FF + 2 SS + 4 SF,$$

which is the 10 proposition of the same book,

Almost the whole doctrine of *proportionals*, viz. permutation, inversion, conversion, composition, division of *Ratios*, and proportion *ex aequo*, and consequently the most useful propositions of the 5th book are clearly demonstrated by one definition, and that is of similar or like parts, which are said to be such as are after the same manner or equally contained in their wholes; thus (in *Fig. 6*) the Antecedents A and C are either equal to their consequents or greater, or less, if equal, the thing is manifest, if less, then (by the definition of *proportionals*) A and C are like parts of B and E, therefore what proportions the whole B and E have to one another, the same will A and C have, which is permutation, likewise $E : C :: B : A$ which is inversion; so also if from *proportionals* you take like parts, the remainders will be proportional, whence conversion and division are demonstrated; and if to proportionals you add like parts, the wholes will still be *proportional*, which is Composition &c. If the *antecedents* be greater than the *Consequents*, the *Consequents* will be like parts of them, and the demonstration exactly the same with the former.

The first of the 6th book is proved by considering the generations of *Parallelograms*, which are produced by drawing or multiplying the *perpendicular* upon the *basis*, that is, taking it so often as there can parts and divisions in the *basis*: therefore (in *Fig. 7*) the same proportion that RX single, hath to NX single, the same hath RX multiplied by XZ, that is, repeated a certain number of times, to NX multiplied by XZ, that is, repeated the same number of times; which is as much as to say $RX : NX :: par : RZ : par : NZ$; now that this proportion also is true in oblique angled *parallelograms*, is proved, because they are equal to rectangled ones upon the same *basis* and between the same *Parallels*, as does thus independently appear (in *Fig. 7*) the triangles RQX and MPZ are equal, for $RX = MZ$, $QX = PZ$, $RM = QP$, therefore
adding

adding to both $MQ, RQ=MP$, if therefore from these equal triangles you take what is common *viz.* MLQ , the remainders will be equal $RXL=QLZP$; to both which add XLZ , and the whole parallelograms will be equal, $RZ=QLZ$ (Q. E. D.) that triangles also having a common *basis*, are in the proportion of their altitudes does hence follow, because they are the halves of *parallelograms* upon the same *basis*, this also is true, and the demonstration exactly the same in prisms, Pyramids, Cylinders, and cones, having the same *basis*.

To prove the 16th of the 6th I suppose (in *Fig. 6.*) the 4 lines A, B, C, E . to be proportional, that is, granting A and C to be the lesser terms, the same way that A is contained in B , so is C in E , and that D is the *denominator* of the *ratio*, 'twill follow then that B is made up of A , multiplied by D , and E of C multiplied by D , so that $AD=B$, and $CD=E$, draw therefore the extremes upon one another, that is A upon CD and the meanes, that is, C upon AD , the factors being the same, I say the products ACD and CAD are the same and consequently equal (Q. E. D.

I know not whether it be worth the while to add somewhat (tho altogether impertinent to this present subject) concerning *Mons: Comier's probleme* which he lately proposed with ostentation enough to all *Mathematicians* to be solved, as if it contained something new, whereas tis no more then the old business of *doubling the Cube* a little disguised, this has been shewn by several, but by none (I think) after the *algebraical way*, or so briefly as follows in *Fig. 3.*

$$A : 2X :: X : 2 \frac{X}{A} q = P \quad \text{per 8. 6.}$$

$$Aq + 2AX \text{ ---- } \frac{2Xc}{A} = 4 \frac{Xqq}{Aq} \quad \text{per 47. 1.}$$

$$Aqq + 2AcX \text{ ---- } 2XcA = 4Xqq$$

$$Aqq + 2AcX = 4Xqq + 2XcA \quad \text{resolving which equation}$$

$$A + 2X : 2A + 4X :: Xc : Ac :$$

into an *Analogy.*

that is, Xc (the *cube* upon X) is $\frac{1}{2}$ of Ac (the *cube* upon A)

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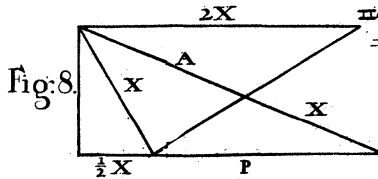
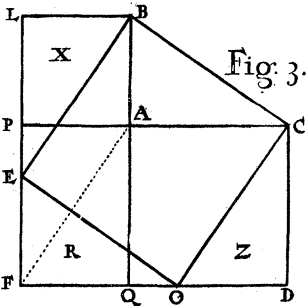
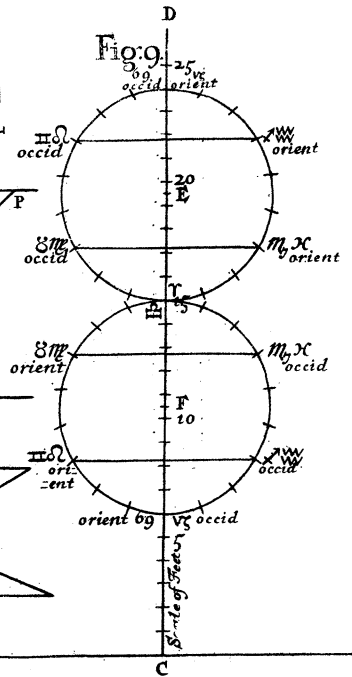
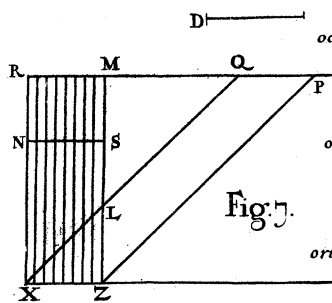
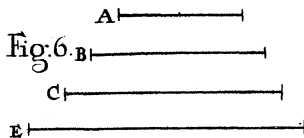
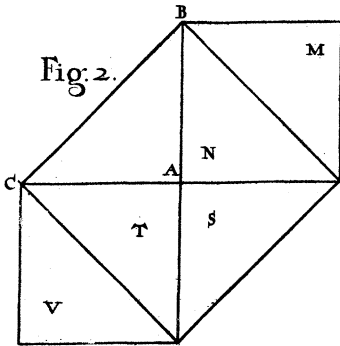
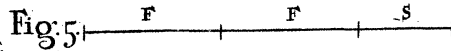
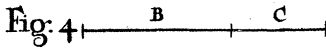
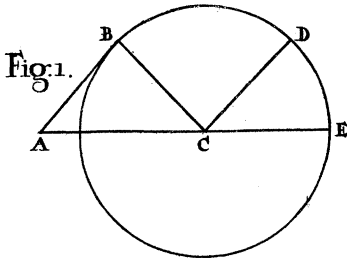


Fig. 10

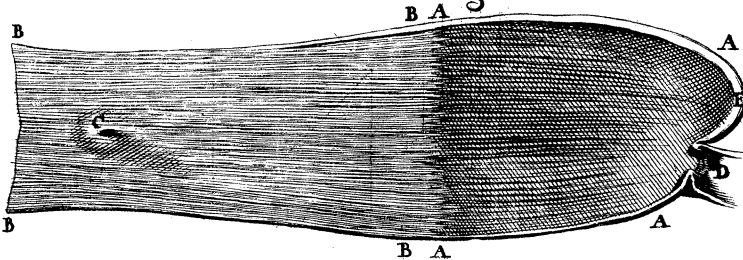
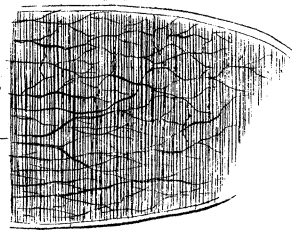
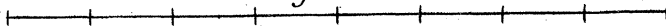


Fig. 11



Scale of Inches



M. Burg. sculp.